

A Back-Reaction Induced Lower Bound on the Tensor-to-Scalar Ratio

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(Dated: February 1, 2008)

There are large classes of inflationary models, particularly popular in the context of string theory and brane world approaches to inflation, in which the ratio of linearized tensor to scalar metric fluctuations is very small. In such models, however, gravitational waves produced by scalar modes cannot be neglected. We derive the lower bound on the tensor-to-scalar ratio by considering the back-reaction of the scalar perturbations as a source of gravitational waves. These results show that no cosmological model that is compatible with a metric scalar amplitude of $\approx 10^{-5}$ can have a ratio of the tensor to scalar power spectra less than $\approx 10^{-8}$ at recombination and that higher-order terms leads to logarithmic growth for r during radiation domination. Our lower bound also applies to non-inflationary models which produce an almost scale-invariant spectrum of coherent super-Hubble scale metric fluctuations.

PACS numbers: 98.80.Cq

I. INTRODUCTION

Direct measurement of primordial gravitational waves or of the tensor to scalar ratio, r , would provide cosmologists with invaluable information about the state of the inflationary universe - specifically, the value of the Hubble rate. The reason for this is that, unlike detecting the scalar amplitude, detecting the tensor amplitude unambiguously determines the value of the inflationary scale since [1]

$$r \approx \frac{m_{Pl}^2}{\pi} \left(\frac{V'}{V} \right)^2, \quad (1)$$

where slow-roll conditions have been assumed. The ratio r is defined as the ratio of the power spectra of tensor to scalar modes.

The current experimental upper bound is $r < 0.5$ and it is anticipated that, in the next few years, we will be able to probe for values of r in the range $10^{-1} - 10^{-2}$ [2]. In the approximation in which both the scalar and tensor spectra are computed in linear cosmological perturbation theory, the value of r can be used to distinguish different classes of inflationary models. Whereas single scalar field models of large field inflation [3] typically predict a value of r of around 10^{-2} (plus or minus an order of magnitude), small field models of inflation [4, 5] with potentials of Coleman-Weinberg type [6] and hybrid inflation models [7] typically predict a very small amplitude. For example, for a potential which in the slow-roll region can be approximated by

$$V = V_0 - \lambda \phi^4, \quad (2)$$

where $\lambda \ll 1$ is a self-coupling constant, the ratio is

$$r \sim \lambda^2 \left(\frac{H}{m_{Pl}} \right)^2. \quad (3)$$

In the case of hybrid inflation, the small value of r can be seen either by direct computation, or else by making

use of the “Lyth bound” [8]

$$r < \frac{8}{N^2} \left(\frac{\Delta\phi}{m_{Pl}} \right)^2, \quad (4)$$

where N is the number of e-foldings of inflation between the time when cosmological scales exist the Hubble radius during inflation and the end of the inflationary phase, and $\Delta\phi$ is the change in the value of ϕ during the corresponding period of inflation (which in the case of hybrid inflation is much smaller than m_{Pl}). In particular, since the power spectrum of gravitational waves is given by the Hubble constant during inflation, low scale models of inflation can have tiny values of r , values as low as 10^{-24} appearing in the literature [9]. Most of the recently popular string-motivated brane-antibrane inflation models (see [10] for recent reviews) are of hybrid inflation type and hence lead to very small values of r , and in this context an upper bound on the primordial tensor to scalar ratio has been proposed [11].

The purpose of this short note is to point out that there exists an alternative mechanism for the production of tensors that is completely independent of the specific inflationary model. The back-reaction of scalar perturbations can act as a source of gravitational waves which, despite not being typically thought of as a main contributor to r in the case of large field inflationary models, could dominate the value of r in other models. The main contribution to the induced gravitational waves is produced after reheating. Hence, our analysis and the resulting lower bound on r applies to all models which produce an almost scale-invariant spectrum of coherent super-Hubble scale fluctuations.

Note that there has been a substantial body of previous work on gravitational waves induced at higher order in perturbation theory [12],[13], in particular on gravitational waves produced at the end of inflation [14, 15]. There has also been recent work on gravitational waves generated by the curvaton [16]. What is new in our work

is the focus on the modifications to the tensor to scalar ratio. Note that the conclusion that secondary gravitational waves may dominate over the primordial ones has also recently been reached in [17].

II. GRAVITATIONAL WAVES FROM SCALARS

Our starting point is the following perturbed line element written in terms of conformal time η , and denoting the scale factor of the background by $a(\eta)$:

$$ds^2 = a(\eta)^2[(1+2\Phi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j]. \quad (5)$$

The scalar metric fluctuations are described by Φ , a function of space and time, and the gravitational waves are given by the transverse and traceless tensor h_{ij} . We are using longitudinal gauge to describe the linearized scalar metric perturbations [34], and are assuming the absence of anisotropic stress (see [19] for a comprehensive review of the theory of cosmological perturbations, and [20] for an introductory overview). For each wavenumber k , there are two polarization states for gravitational waves, each described by an amplitude h and a polarization tensor ϵ_{ij} .

In other words, each second rank tensor h_{ij} can be decomposed as

$$h_{ij}(\eta, x) = h_{(1)}(\eta, x)\epsilon_{ij}^{(1)} + h_{(2)}(\eta, x)\epsilon_{ij}^{(2)}. \quad (6)$$

In what follows, we make use of this decomposition to express the dynamics of the gravitational waves, h_{ij} , in terms of the scalar coefficient functions $h^{(1,2)}$.

At linear order in the fluctuations, the scalar and tensor modes are independent. However, if the magnitude of the linearized gravitational waves is very small compared to the magnitude of the scalar modes, then the secondary tensor fluctuations generated by non-linear interactions from the scalar modes cannot be neglected. We can determine the coupling between Φ and h_{ij} by expanding the gravitational action in powers of the perturbation amplitude. At leading order, we find:

$$\begin{aligned} S_{grav} &= \frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} R \\ &= \frac{m_{Pl}^2}{16\pi} \int d^4x \left[\frac{a(\eta)^2}{2} ((h')^2 - (\partial_i h)^2) \right. \\ &\quad \left. - a^2(\eta) h (8\Phi \nabla^2 \Phi + 6(\partial_i \Phi)^2) + \dots \right]. \end{aligned} \quad (7)$$

In the above (and below), we have suppressed the indices denoting the graviton polarization state as both states couple identically to matter.

Additional sources for gravitons arise from the matter sector coupled through the kinetic term. In the spirit of simple inflationary universe models, we will take the matter sector to be described by a single canonically nor-

malized scalar field ϕ . In this case we write

$$\begin{aligned} S_{matter} &= \int d^4x \frac{1}{2} \sqrt{-g} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)) \quad (8) \\ &= \int d^4x \dots - \frac{1}{2} a^2(\eta) h [(\partial_i \delta\phi)^2 \\ &\quad + 2\Phi \partial^i \phi \partial_i \delta\phi + 2\Phi^2 (\partial_i \phi)^2] \dots, \end{aligned}$$

where we have extracted only the terms which are linear in h . If the background is homogeneous, then the terms proportional to $\partial_i \phi$ are absent.

Combining (7) and (8) leads to the following, sourced equation of motion for the tensors:

$$\begin{aligned} \mu'' + (-\nabla^2 - \frac{a''}{a})\mu &\quad (9) \\ &= \frac{1}{3} a(\eta) [16\Phi \nabla^2 \Phi + 12(\partial_i \Phi)^2 \\ &\quad + \frac{16\pi}{m_{Pl}^2} (\partial_i \delta\phi)^2], \end{aligned}$$

where we have rescaled the tensor mode, $h = \mu/a(\eta)$. This transformation has the advantage of eliminating the Hubble damping term and making the particle production process more transparent.

In the standard case, squeezing of the quantum vacuum state leads to particle production [21] with the squeezing being determined by the dynamics of the background. The squeezing plays the role of a tachyonic instability, which can be seen by the presence of a negative mass term in (9). Exactly the same thing happens in the scalar sector except that, instead of the squeezing term being proportional to $-\frac{a''}{a}$, it is proportional to $-\frac{z''}{z}$ with $z = a\frac{\phi'}{H}$. It is this different dependence on the behaviour of the inflaton that makes it possible to have a scenario in which the tensor amplitude is far below that of the scalar.

We now consider a scenario in which the squeezing produces a negligible amount of gravitons, but a large amplitude of scalar metric fluctuations. In this case, we can determine the amplitude of the (secondary) tensors being produced via interactions from the scalar modes (in the language of particle physics, one could say “by scalar decay”). To this end, we neglect the squeezing term in (9) entirely. Since we focus on super-Hubble scale fluctuations, we can also neglect the spatial gradient terms [35]. With these approximations, the partial differential equation (9) reduces to an ordinary one which takes the form

$$\begin{aligned} \mu'' &= \frac{1}{3} a(\eta) [16\Phi \nabla^2 \Phi + 12(\partial_i \Phi)^2 \\ &\quad + \frac{16\pi}{m_{Pl}^2} (\partial_i \delta\phi)^2]. \end{aligned} \quad (10)$$

The above equation (10) makes it clear that scalar metric fluctuations source gravitational waves at all times. We can simplify the equation by noting that the scalar sector has only one independent degree of freedom [19].

During slow-roll inflation we can make use of the constraint equation [22]

$$\delta\varphi \simeq -2\frac{V}{V'}\Phi, \quad (11)$$

to recast (10) as

$$\begin{aligned} \mu'' \simeq & \frac{4}{3}a(\eta)[4\Phi\nabla^2\Phi \\ & + (3 + \frac{16\pi}{m_{Pl}^2}(\frac{V}{V'})^2)(\partial_i\Phi)^2]. \end{aligned} \quad (12)$$

After reheating, a formula analogous to (10) but with matter taken to be a perfect fluid must be used.

We first focus on the gravitational waves generated during inflation. Making use of the definition of the slow-roll parameter

$$\epsilon \equiv \frac{m_{Pl}^2}{16\pi}(\frac{V'}{V})^2 \quad (13)$$

in (12), we see something perhaps a little surprising: the second term in the source (the term originating from the matter sector) scales as $1/\epsilon$. As mentioned above, in the standard case (ignoring back-reaction), a small value of ϵ suppresses the amplitude of the tensors. However, including the effects of back-reaction, we see that a small ϵ leads to a higher amplitude of induced gravitational waves.

We now wish to use the above formalism to compute the amplitude of the secondary gravitational waves at the time when the mode of interest is crossing the Hubble radius during the initial inflationary phase, assuming that the initial state of all fluctuation modes on ultraviolet scales is the quantum vacuum state. Because of our assumption on the initial state, it is only scalar modes which have already exited the Hubble radius and thus undergone some squeezing which can produce such secondary gravitational waves.

One way to compute the source term in (12) is to expand Φ in Fourier modes, insert into the expression on the right hand side of the equation, use the fact that the scalar spectrum is scale-invariant on super-Hubble scales, and to extract the k 'th Fourier mode of μ by inverse Fourier transform of the result and by contracting with the polarization tensor of the gravitational wave. The integrals over momenta run from an infrared cut-off (which can be taken to be the scale corresponding to the Hubble radius at the beginning of inflation, but which does not play any role in this calculation) to momenta corresponding to the Hubble radius. Modes on sub-Hubble scales are in their vacuum state and hence cannot generate any gravitational waves. Also, as shown in [15], the effect of short wavelength scalar fluctuations would average to zero in our coupling term.

A short cut to the calculation is to simply to promote Φ to the status of quantum operator [19] and to take the vacuum expectation value of the right hand side of (12),

making use of the two-point function (in the limit that $r \rightarrow 0$)

$$\langle 0|\Phi(x,\eta)\Phi(x+r,\eta)|0\rangle = \int_0^\infty \frac{dk}{k} \frac{\sin(kr)}{kr} |\delta_\Phi|^2, \quad (14)$$

and restricting the integral to the IR phase space, $k = [0, \mathcal{H}]$, where \mathcal{H} is the Hubble rate in conformal time. In the above, $|\delta_\Phi|^2$ is the power spectrum of the scalar modes (including the phase space factor k^3). Multiplying with the polarization tensor to extract the amplitude μ of the gravitational wave, we thus get

$$\mu'' \simeq -\frac{2}{3}a(\eta)[7 + \frac{1}{\epsilon}]\mathcal{H}^2|\delta_\Phi|^2, \quad (15)$$

through which the evolution of μ is now directly related to the scalar power spectrum $|\delta_\Phi|^2$. Note that in the above formula, the scalar metric power spectrum is the one during inflation.

Several remarks need to be made about this calculation: to obtain this result, we assumed scale-invariance of the scalars. As well, (15) can only be used to extract information about μ on large scales, i.e. $H \gg k_{phys}$. The physical reason for this condition is obvious - momentum conservation forces the produced tensors to have smaller momenta than the scalars from which they originate. Considering contributions arising solely from the scalar IR sector is not as limiting as it may seem due to the fact, through the course of inflation, the IR phase space grows exponentially while the UV phase space remains constant (see the arguments presented in [22] and [23]). However, in our calculation there is no IR divergence and the non-gradient contributions of the IR modes (those which in the case of adiabatic matter fluctuations are pure gauge [24, 25, 26]) do not contribute. Note that the $\frac{1}{\epsilon}$ dependence on the size of the effects of back-reaction has also been noticed in [12], [27], and in a very different context,[28].

Integrating (15) immediately yields

$$\mu \simeq -\frac{a(\eta)}{3}[7 + \frac{1}{\epsilon}]|\delta_\Phi|^2, \quad (16)$$

or

$$h \simeq -\frac{1}{3}[7 + \frac{1}{\epsilon}]|\delta_\Phi|^2, \quad (17)$$

where we have used $a(\eta) = \frac{-1}{H\eta}$, and $\mathcal{H} = a(\eta)H$. The scale invariance of the above is manifest.

The amplitude of the power spectrum of scalar metric perturbations during inflation is suppressed compared to the spectrum after inflation by the factor ϵ^2 , the inverse of the squeezing factor of Φ^2 during the transition between the inflationary phase and the radiation phase of Standard Cosmology. Taking the current power spectrum to be $\approx 10^{-10}$, we estimate the contribution to the tensor-scalar ratio induced by back-reaction before reheating to be

$$r \approx 10^{-10}\epsilon^2. \quad (18)$$

This is negligible compared to the contribution to r of the linear gravitational waves produced from quantum vacuum fluctuations.

Next, we consider the gravitational waves induced by back-reaction after reheating. To obtain a lower bound on the effects of back-reaction, we neglect the matter contribution on the right hand side of (10). Hence, there is no ϵ^{-1} enhancement factor. However, the amplitude of Φ to be used is not suppressed by ϵ . Hence, repeating the steps which led to (17) leads to an analogous formula without the ϵ^{-1} term in the square parentheses on the right-hand side of the equation. This gives a contribution

$$r \approx 10^{-10}. \quad (19)$$

Hence, the back-reaction of scalar metric fluctuations has a more important effect after reheating than before.

Our analysis is not restricted to the inflationary era. Interestingly, (15), predicts the creation of gravitons on super-Hubble scales even after reheating. Direct substitution of the radiation-domination scale factor ($a(\eta) = \eta/\eta_0$) leads to the result

$$h(\eta) \sim \ln(a(\eta)). \quad (20)$$

At the onset of matter domination ($a(\eta) \sim \eta^2$), the amplitude of the super-Hubble tensor modes again becomes constant.

Taking the above into consideration, we take as our final estimate for the back-reaction determined tensor-to-scalar ratio at the time of recombination to be

$$r \approx 10^{-8}. \quad (21)$$

At the order in perturbation theory we are now working in we must also consider the decay of the gravitational waves into scalars through interaction terms of the form

$$S_{h^2 \rightarrow \Phi} \propto \int d^4x a^2(\eta) \Phi h_j^i \nabla^2 h_i^j. \quad (22)$$

However, for the values of r obtained here, this decay is negligible.

III. CONCLUSION

We have estimated the amplitude of gravitational waves produced via the back-reaction of scalar metric

fluctuations, focusing on their effect on the tensor-to-scalar ratio r . In conclusion, our result (19) shows that back-reaction of scalar modes will produce a sizeable gravitational wave background in a wide variety of inflationary models in which the value of r computed from purely linear theory is very small.

The contribution to r from back-reaction of scalar modes picks up its major contribution after reheating. Hence, our lower bound on r will be valid in any theory producing a roughly scale-invariant spectrum of fluctuations at late times, provided these fluctuations are coherent outside the Hubble radius (which appears to be demanded by the data based on the acoustic peak structure of the observed angular power spectrum of cosmic microwave anisotropies). Examples of such theories are the Ekpyrotic universe scenario [29], the Pre-Big-Bang model [30] and the recently proposed string gas structure formation paradigm [31]. Our analysis is particularly relevant for the Ekpyrotic scenario in which a negligible amount of gravitational waves are generated in the linear theory. Back-reaction, however, will induce a contribution which is similar in magnitude to what is induced in many small field inflationary models.

An interesting point that was noted was the ability of backreaction to increase the amplitude of IR modes after inflation. This is in stark contrast with the linear result in which the amplitudes of both scalar and tensor modes are frozen until the time of second Hubble crossing.

This work is complementary to an earlier study of the back reaction effects of scalar modes onto the scalar modes themselves [32] which were shown to be negligible. As this note was being finalized, a paper appeared [33] which studies vector modes produced by primordial density fluctuations.

Acknowledgments

The authors would like to thank Aaron Berendsen for useful discussion. We are grateful to Cristian Armendariz-Picon for pointing out an error in an earlier version of this manuscript. This work was supported by an NSERC Discovery Grant, by the Canada Research Chairs program and by funds from a FQRNT Team Grant.

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 - [35] A formalism which does not make these two approximations and thus applies also to short wavelength fluctuations was recently discussed in [15]. The full partial

differential equation can be solved by a Greens function method. The solution involves an integral over momenta and time. Applied to oscillatory fluctuations, the time integral produces a delta function which leads to the conclusion that short wavelength scalar modes do not contribute to the production of gravitational waves. For long wavelength fluctuations, this delta function does not appear, and, as we show below, gravitational waves are indeed generated. One way of convincing oneself that this must be possible is that the coupling we are analyzing is that between two vectors (the gradients of the scalar field) and a tensor, and this coupling in general does not vanish. However, the analysis of [15] gives us a further argument to focus exclusively on super-Hubble modes.